

San Francisco Unified School District Public School Enrollment Fair

The Enrollment Process: Diversity Index Lottery Explained

The lottery used as a part of San Francisco Unified School District's (SFUSD) assignment process uses a measure of qualitative variation referred to generically as a "diversity index" (DI). This measure of diversity generally describes the probability that two individuals selected at random from a population will be different from one another in terms of an identified characteristic or set of characteristics.

There are seven primary steps of the Diversity Index process in SFUSD: (1) defining diversity statistically; (2) identifying the relevant diversity characteristics; (3) creating a student profile based on these characteristics; (4) identifying each school's applicant pool; (5) calculating a composite initial diversity score for each school and grade level; (6) assessing the impact of each student's individual profile on the composite score for each school and grade level in order to fill available seats through the lottery process and (7) reconciling tentative assignments for applicants selected to fill multiple seats in order to give each applicant a single offer.

In SFUSD's implementation of the Diversity Index, there are five characteristics for non-kindergarten applicants and four characteristics for kindergarten applicants: Academic Achievement Score (AAS), Socioeconomic Status (SES), Extreme Poverty (EP), Home Language (HL) and – for non-kindergarten applicants – API Rank of Sending School (ARSS). Each characteristic chosen by SFUSD is used in a binary form, so an applicant will have either a "1" or "0" for each of the four or five characteristics used. Because there are five binary characteristics, there are 32 possible profiles for non-kindergarten applicants.

An initial evaluation of the diversity of any applicant pool will look at the ratio of the characteristics present in that pool. For example, in a pool in which 75 applicants have a particular characteristic (and thus each receive a "1" for that characteristic) and 25 do not possess that characteristic (and thus receive a "0" for that characteristic), there is a

75% chance on the first draw that a positive applicant (*i.e.*, one with a “1” for that characteristic) will be drawn. Because the value of each characteristic is binary, the optimal ratio is always 50/50. When using multiple binary characteristics to describe diversity, as is currently the case in SFUSD, the optimal diversity of a group would be expressed by an equal number of applicants from each profile possible, resulting in a 50/50 ratio for each of the four (for kindergarten) or five (for non-kindergarten) characteristics under consideration.

A common misunderstanding with respect to the Diversity Index process is the assumption that there is a “perfect” or “ideal” profile based on some kind of additive properties associated with the diversity characteristics – for example, that having all “1s” is better than having all “0s”. There is, however, no perfect profile since the lottery program is always attempting to balance the ratios of a given characteristic among many characteristics. Rather, there are only profiles that bring these ratios closer or farther from the optimal 50/50 balance at a given evaluation point. In other words, a profile that would, at one point during the lottery process, pull the ratio further away from the 50/50 ratio may later in the lottery process be the profile that best pulls the selected population toward the 50/50 goal.

Lottery Simulation

Following this description are a number of attachments that describe in detail the Lottery process as they could apply to the selection of applicants for a hypothetical 6th grade.

Attachment A provides a brief narrative of the seven steps of the Diversity Index process.

Attachment B provides the 32 various possible profile combinations for grades above kindergarten.

Attachment C provides the distribution of sixth-grade applicants for 200 seats at an example school. Of the 465 total applicants, 24 have Profile #1 (that is, they have a “1” on all five diversity characteristics.) Thirteen of the applicants have Profile #2 (that is,

they have a “0” on all five characteristics). The most common profile is Profile #24, with 82 applicants sharing this set of characteristics. In contrast, in this example, no students at all have 7 of the 32 possible profiles.

Attachment D shows the students pre-assigned to that class at the same sample school. The pre-assigned students include siblings and students placed in special programs. These students are placed before the lottery process occurs. Attachment D shows that there were 23 siblings and 22 program placements in the sixth grade at this school, filling a total of 45 seats. The attachment also shows the diversity profile of each of these students: Six have Profile #1; two have Profile #2; and again Profile #24, with eight students, is the most common profile.

Attachment E calculates the ratios to be used in the lottery.

Attachment F shows the calculation of the Composite Diversity Index (“CDI”) of the pre-assigned students in the sixth grade in the example school. For each characteristic, the Characteristic CDI is calculated by (1) calculating the proportions of students with and without that characteristic; (2) squaring those two numbers; and (3) subtracting the sum of the squares from one. For example, for Academic Achievement Status: (1) 23 of 45 students, or 0.51, had a “0” and 22 of 45 students, or 0.49, had a “1”; (2) 0.51 squared is 0.2601, and 0.49 squared is 0.2401; and (3) one minus the sum of those squares is 0.4998. Accordingly, the Characteristic CDI for Academic Achievement Status is 0.4998. The Total Composite Diversity Index is the average of the Characteristic CDI for each characteristic. In this example, the Total Composite Diversity Index, taking the average of the Characteristic CDIs for each of the five characteristics, is 0.4607.

The next three attachments show the effect of assigning a student with each of three different individual profiles on the CDI of the sixth-grade class at the example school. Attachment G shows how assigning a student with Profile #1 would affect the Total Composite Diversity Index score for the sixth-grade class. In this case, it would increase the score from 0.4607 to 0.4614, making the class slightly more diverse.

Attachment H shows that assigning an applicant with Profile #18 would make the sixth-grade class slightly less diverse, lowering its Total Composite Diversity Index to 0.4603.

Attachment I calculates the impact of assigning an applicant with yet a different profile, with the effect again increasing the Total Composite Diversity Index for the class.

Attachment J shows how the computer would select the applicant that would best improve the diversity of the sample class to fill the 46th seat in that class (the first seat available after the sibling and program placements, as discussed above). After calculating the effect on the Total Composite Diversity Index of students with each of the available profiles, the lottery program ranks the applicants in terms of their positive impact on diversity, from greatest to lowest positive impact. The applicant with the highest-ranked profile – in this example, the applicant with Profile #1 – is selected because that applicant has the greatest positive impact on the diversity of the class. Then, a new composite Diversity Index would be calculated and the process would be repeated until all the available seats in the sample class are filled.

Attachment K indicates how a final offer is reconciled from multiple tentative assignments for the applicant with Profile #1.

Attachment A: Diversity Index Process

Stage A: Defining diversity statistically

The Diversity Index (**DI**) is a formula that calculates the probability of selecting two individuals from a population that will be different on **one** pre-defined characteristic.

The Composite Diversity Index (**CDI**) is a formula that calculates the probability of selecting two individuals from a population that will be different on a **number of pre-defined characteristics**.

Both indexes produce a probability score that varies from 0 to 1 (i.e., if stated in percentages, zero percent to 100 percent chance of occurring)

Therefore, the higher the score (i.e., the higher the probability of selecting two people who are different) means a more diverse group.

Stage B: Defining diversity characteristics

Academic Achievement Status (AAS)

Extreme Poverty (EP)

Socioeconomic Status (SES)

Home Language (HL)

API Rank of Sending School (ARSS)

Stage C: Defining student profiles

All five characteristics make up a student's profile for 1st through 12th graders; four for KG, all except ARSS

The profile is the unique set/combination of 1's and 0's for each diversity characteristic for all diversity characteristics

[Example: There are 32 unique combinations for 1st through 12th graders; there are 16 unique combinations for KG]

Stage D: Defining a school's applicant pool

A school's applicant pool is made up of all students who request that school

Each applicant pool will be comprised of: Sibling, Program Placement, Attendance Area, and Non-Attendance Area requests

Each applicant pool will have a different distribution of student profiles

[Example: There are 712 students in Middle School XXX's application pool; **200 seats are available to be filled**]

Stage E: Determining a grade level's initial diversity score and profile distribution

A grade level's initial diversity is determined by the pre-assignment of **sibling and program requests**

[Example: There are 74 sibling and program assignments which make up the initial grade level diversity]

Each of their profile characteristics are added together to generate a group profile distribution for that grade level

[Example: Each profile is weighted by the number of applicants having that profile and then the 0's and 1's are summed for each characteristic]

Calculate the composite diversity index for the grade level

[Example: CDI is calculated for the 74 seats that have been assigned to that grade level; **126 remain to be assigned**]

Stage F: Assessing each profile's impact on the initial grade level composite diversity index score in order to fill remaining seats

For alternative schools:

Each applicant is evaluated for the next available seat by calculating the impact of each profile remaining in the applicant pool and assigning a rank to that profile for the available seat.

[Examples: CDI's are calculated for profile #'s 1,52, and 42 for the 75th seat]

For the 75th seat, all profiles are ranked from highest CDI to lowest CDI; the profile with the highest CDI is selected to fill the 75th seat

[Example: Ranking of profile CDI scores to fill 75th seat]

If there are ties within a profile; school preference rank gets first priority; if ties remain, then random selection.

For regular schools:

Each applicant that resides in the school's attendance area is evaluated for the next available seat by calculating the impact of each profile remaining in the applicant pool and assigning a rank to that profile for the available seat.

Attendance area applications are given priority and assigned first as long as positive CDI scores exist for any profiles

If there are ties within a profile; school preference rank gets first priority; if ties remain, then random selection.

[Examples: CDI's are calculated for profile #'s 1,52, and 42 for the 75th seat]

For the 75th seat, all profiles are ranked from highest CDI to lowest CDI; the profile with the highest CDI is selected to fill the 75th seat

[Example: Ranking of profile CDI scores to fill 75th seat]

When the CDI is negative for all attendance area profiles, then the non-attendance area students are added

If there are ties within a profile; attendance area gets first priority; school preference rank gets second priority; if ties remain, then random selection.

Once the 75th seat is filled, the grade level profile distribution is updated to reflect the new assignment and new base CDI score

This process is repeated until all remaining seats are filled.

Stage G: Reconciling temporary assignments for a single offer

All applicants that are selected to fill more than one seat will be given the offer to the school that was ranked highest on their application

Attachment B: Profile Combinations for five characteristics

Profile #	AAS	SES	EP	HL	ARSS	Profile Mix	
1	1	1	1	1	1	5 ones	0 zeros
2	0	0	0	0	0	5 zeros	0 ones
3	0	1	1	1	1	4 ones	1 zero
4	1	0	1	1	1		
5	1	1	0	1	1		
6	1	1	1	0	1		
7	1	1	1	1	0		
8	1	0	0	0	0	4 zeros	1 one
9	0	1	0	0	0		
10	0	0	1	0	0		
11	0	0	0	1	0		
12	0	0	0	0	1		
13	0	0	1	1	1	3 ones	2 zeros
14	1	0	0	1	1		
15	1	1	0	0	1		
16	1	1	1	0	0		
17	0	1	0	1	1		
18	1	0	1	0	1		
19	1	1	0	1	0		
20	0	1	1	0	1		
21	1	0	1	1	0		
22	0	1	1	1	0		
23	1	1	0	0	0	3 zeros	2 ones
24	0	1	1	0	0		
25	0	0	1	1	0		
26	0	0	0	1	1		
27	1	0	1	0	0		
28	0	1	0	1	0		
29	0	0	1	0	1		
30	1	0	0	1	0		
31	0	1	0	0	1		
32	1	0	0	0	1		

Totals (1's) 16 16 16 16 16
 Totals (0's) 16 16 16 16 16

Attachment C: Middle School XXX 6th Grade Applicant Pc

**200 seats available
of applicants**

Profile #	AAS	SES	EP	HL	ARSS		
1	1	1	1	1	1		24
2	0	0	0	0	0		13
3	0	1	1	1	1		2
4	1	0	1	1	1		4
5	1	1	0	1	1		35
6	1	1	1	0	1		22
7	1	1	1	1	0		14
8	1	0	0	0	0		75
9	0	1	0	0	0		5
10	0	0	1	0	0		8
11	0	0	0	1	0		12
12	0	0	0	0	1		8
13	0	0	1	1	1		1
14	1	0	0	1	1		0
15	1	1	0	0	1		5
16	1	1	1	0	0		0
17	0	1	0	1	1		14
18	1	0	1	0	1		56
19	1	1	0	1	0		23
20	0	1	1	0	1		6
21	1	0	1	1	0		0
22	0	1	1	1	0		1
23	1	1	0	0	0		14
24	0	1	1	0	0		82
25	0	0	1	1	0		12
26	0	0	0	1	1		18
27	1	0	1	0	0		9
28	0	1	0	1	0		0
29	0	0	1	0	1		0
30	1	0	0	1	0		0
31	0	1	0	0	1		2
32	1	0	0	0	1		0
Total							465

Attachment D: Pre-Assignemnts of sibling and program placement applicants - the initial count

Profile #	AAS	SES	EP	HL	ARSS	Sibling	Program	Initial Count	Total Pool
1	1	1	1	1	1	1	5	6	24
2	0	0	0	0	0	2		2	13
3	0	1	1	1	1				2
4	1	0	1	1	1				4
5	1	1	0	1	1	2		2	35
6	1	1	1	0	1	1	2	3	22
7	1	1	1	1	0	2		2	14
8	1	0	0	0	0	2		2	75
9	0	1	0	0	0	4		4	5
10	0	0	1	0	0	1	2	3	8
11	0	0	0	1	0	1		1	12
12	0	0	0	0	1				8
13	0	0	1	1	1				1
14	1	0	0	1	1				0
15	1	1	0	0	1				5
16	1	1	1	0	0				0
17	0	1	0	1	1				14
18	1	0	1	0	1	2	5	7	56
19	1	1	0	1	0	1		1	23
20	0	1	1	0	1		1	1	6
21	1	0	1	1	0				0
22	0	1	1	1	0				1
23	1	1	0	0	0				14
24	0	1	1	0	0	2	6	8	82
25	0	0	1	1	0	2	1	3	12
26	0	0	0	1	1				18
27	1	0	1	0	0				9
28	0	1	0	1	0				0
29	0	0	1	0	1				0
30	1	0	0	1	0				0
31	0	1	0	0	1				2
32	1	0	0	0	1				0
Total						23	22	45	465

Attachment E: Profile distribution of Pre-Assignment Students

Profile #	Initial Profiles					Initial Count	Weighted Profiles					ARSS
	AAS	SES	EP	HL	ARSS		AAS	SES	EP	HL		
1	1	1	1	1	1	6	6	6	6	6	6	
2	0	0	0	0	0	2	0	0	0	0	0	
5	1	1	0	1	1	2	2	2	0	2	2	
6	1	1	1	0	1	3	3	3	3	0	3	
7	1	1	1	1	0	2	2	2	2	2	0	
8	1	0	0	0	0	2	2	0	0	0	0	
9	0	1	0	0	0	4	0	4	0	0	0	
10	0	0	1	0	0	3	0	0	3	0	0	
11	0	0	0	1	0	1	0	0	0	1	0	
18	1	0	1	0	1	7	7	0	7	0	7	
19	1	1	0	1	0	1	1	1	0	1	0	
20	0	1	1	0	1	1	0	1	1	0	1	
24	0	1	1	0	0	8	0	8	8	0	0	
25	0	0	1	1	0	3	0	0	3	3	0	
Total						45						

	AAS	SES	LP	HL	ARSS
Total 0's	22	18	12	30	26
Total 1's	23	27	33	15	19

Attachment F: CDI Calculation of Pre-Assigned Applicants

Characteristic CDI is One minus the sum of the proportions squared

Total composite CDI is the sum of the Characteristic CDI divided by the number of characteristics

FACTOR	NUMBER	PROPORTION	GRADE LEVEL CDI
AAS0	23	0.51	$.51 * .51 = .2601$
AAS1	22	0.49	$.49 * .49 = .2401$
	45		$1 - (.2601 + .2401) = 0.4998$
SES0	18	0.40	$.40 * .40 = .1600$
SES1	27	0.60	$.60 * .60 = .3600$
	45		$1 - (.1600 + .3600) = 0.4800$
EP0	12	0.27	$.27 * .27 = .0729$
EP1	33	0.73	$.73 * .73 = .5329$
	45		$1 - (.0729 + .5329) = 0.3942$
HLO	30	0.67	$.67 * .67 = .4489$
HL1	15	0.33	$.33 * .33 = .1089$
	45		$1 - (.4489 + .1089) = 0.4422$
ARSS0	26	0.58	$.58 * .58 = .3364$
ARSS1	19	0.42	$.42 * .42 = .1764$
	35		$1 - (.3364 + .1764) = 0.4872$
COMPOSITE			$(.4998 + .4800 + .3942 + .4422 + .4872) / 5$
			0.4607

Attachment G: Profile #1 CDI Calculation

FACTOR	PROFILE	NUMBER	PROPORTION	CDI with Applicant of Profile #1	INITIAL GRADE CDI	DIFFERENCE	WEIGHT
AAS0		23	0.50				
AAS1	1	23	0.50				
		46		0.5000	0.4998	0.0002	20
SES0		18	0.39				
SES1	1	28	0.61				
		46		0.4764	0.4800	-0.0036	-363
EPO		12	0.26				
EP1	1	34	0.74				
		46		0.3856	0.3942	-0.0086	-857
HLO		30	0.65				
HL1	1	16	0.35				
		46		0.4537	0.4422	0.0115	1149
ARSS0		26	0.57				
ARSS1	1	20	0.43				
		46		0.4915	0.4872	0.0043	429
COMPOSITE				0.4614	0.4607	0.0038	-38

Attachment I: Profile #42 CDI Calculation

FACTOR	PROFILE	NUMBER	PROPORTION	CDI with Applicant of Profile #42	INITIAL GRADE CDI	DIFFERENCE	WEIGHT
AAS0		23	0.50				
AAS1	1	23	0.50				
		46		0.5000	0.4998	0.0002	200
SES0	1	19	0.41				
SES1		27	0.59				
		46		0.4849	0.4800	0.0049	4877
EPO		12	0.26				
EP1	1	34	0.74				
		46		0.3856	0.3942	-0.0086	-8567
HLO	1	31	0.67				
HL1		15	0.33				
		46		0.4395	0.4422	-0.0027	-2691
ARSS0		26	0.57				
ARSS1	1	20	0.43				
		46		0.4915	0.4872	0.0043	4293
COMPOSITE				0.4603	0.4607	-0.0019	-1888

Attachment H: Profile #20 CDI Calculation

FACTOR	PROFILE	NUMBER	PROPORTION	CDI with Applicant of Profile #20	INITIAL GRADE CDI	DIFFERENCE	WEIGHT
AAS0		23	0.50				
AAS1	1	23	0.50				
		46		0.5000	0.4998	0.0002	200
SES0		18	0.39				
SES1	1	28	0.61				
		46		0.4764	0.4800	-0.0036	-3629
EPO		12	0.26				
EP1	1	34	0.74				
		46		0.3856	0.3942	-0.0086	-8567
HLO	1	31	0.67				
HL1		15	0.33				
		46		0.4395	0.4422	-0.0027	-2691
ARSS0		26	0.57				
ARSS1	1	20	0.43				
		46		0.4915	0.4872	0.0043	4293
COMPOSITE				0.4586	0.4607	-0.0104	-10394

Attachment J: Ranked CDI scores for the 46th seat

Profile #	Unranked CDI score
1	0.4614
2	0.45
5	0.45
6	0.45
7	0.45
8	0.45
9	0.45
10	0.45
11	0.45
18	0.4603
19	0.45
20	0.4586
24	0.45
25	0.45

Profile #	Ranked CDI score
1	0.4614 < <u>Selected</u>
18	0.4603
20	0.4586
2	0.45
5	0.45
6	0.45
7	0.45
8	0.45
9	0.45
10	0.45
11	0.45
18	0.45
19	0.45
20	0.45

Attachment K: Reconciling Multiple Tentative Assignments

If the applicant with Profile #1 is offered more than one school of her or his choice the Lottery will select the school choice that was ranked highest on his or her application form as the school to offer the student at the end of the lottery.

Tentative Assignment Results

Choice 1	Sch XXX	Assigned
Choice 2	SchYYY	Not Assigned
Choice 3	Sch BBB	Assigned
Choice 4	Sch CCC	Not Assigned
Choice 5	Sch LLL	Assigned
Choice 6	Blank	
Choice 7	Blank	

Final Offer = Sch XXX the applicant's highest ranked choice that resulted in an assignment